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Piezoresistance properties of quasicrystals

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Abstract
Piezoresistance properties of quasicrystals due to both phonon and phason stresses are investigated. The classical formulae of the piezoresistance effect in crystals are generalized to the case of quasicrystals. The number of independent components of the piezoresistivity tensor and their matrix forms are determined for three-dimensional icosahedral quasicrystals and all two-dimensional quasicrystals with fivefold, eightfold, tenfold and twelvefold symmetries. Our results show that the piezoresistance effect may be related only to phonon stress in the case of dodecagonal quasicrystals or to both phonon and phason stresses in the other case.

1. Introduction

Since the discovery of quasicrystals (QCs) extensive theoretical and experimental studies have been carried out on their structure, stability and elasticity, and have brought about fruitful results [1, 2]. Recently, the literature devoted to other physical properties of QCs has been rapidly growing [3]. One of the outstanding physical properties of QCs is their anomalous resistance property. The resistivity of several QCs is remarkably large, the values of which are larger than those observed in some systems on the insulating side of a metal–insulator transition [4]. In highly resistive and highly ordered QCs the resistance increases with decreasing temperature [5] or with improved order [6], which is a semiconductor-like or semimetallic behaviour. On the other hand, the piezoresistance effect is an important one which consists in the variation of the electrical resistance of a material under the action of mechanical stresses (or strains). The piezoresistance effect is found in many substances, including metals [7] and semiconductors [8]. And the piezoresistance effect is more remarkable in semiconductors than other materials, which can give important direct experimental information about the structure of the energy bands of semiconductors and allows the use of some semiconducting materials for the manufacture of highly sensitive strain gauges which transform

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mechanical strains (stresses) into electrical quantities. In elastic properties the most particular feature that distinguishes QCs from ordinary crystals is that there are two types of low-energy elastic (hydrodynamic) excitations—phonons and phasons [9]. It is natural for one to ask what piezoresistance properties would be expected in QCs. What characteristic feature could follow from their symmetries alone and not from the details of atomic position and interatomic interactions? We think it is worthwhile to propose a theoretical insight into it. This is the purpose of this paper. We will explore the piezoresistance properties of three-dimensional (3D) icosahedral QCs and all two-dimensional (2D) QCs with crystallographically forbidden symmetries [10]. It is found that the piezoresistive behaviour of QCs is more complicated than that of ordinary crystals because of the presence of the phason field. The number of independent components of the piezoresistivity tensor and their matrix forms are determined for 3D and 2D QCs with fivefold, eightfold, tenfold and twelvefold symmetries. All results are given in section 3. Conclusions are given in section 4.

2. Piezoresistance effect in QCs

It is well known that the appearance of the ordinary phonon and additional phason degrees of freedom in the hydrodynamics for QCs lead to two kinds of strain field, the phonon strain $E_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j)$ and the phason strain $W_{\alpha i} = \partial_i w_\alpha$ as well as two kinds of stress field: the phonon stress T_{ij} and the phason stress $H_{\alpha i}$ [11], where subscripts $i, j, k, \ldots$ are used for coordinate components in the physical space, and subscripts $\alpha, \beta, \gamma, \ldots$ for coordinate components in the perpendicular space. Thus, in analogy with the case of conventional crystals [12, 13] the piezoresistance effect can be represented as a change of the resistivity tensor ρ_{ij} under the action of stresses T_{mn} and $H_{\alpha n}$:

$$\mathcal{E}_i = (\rho^0_{il} + P^{(1)}_{ilmn} T_{mn} + P^{(2)}_{il\alpha n} H_{\alpha n}) J_l = (\rho^0_{il} + \rho^{(1)}_{il} + \rho^{(2)}_{il}) J_l, \tag{1}$$

where $\mathcal{E}$ is the electric field intensity, $\mathbf{J}$ is the current density, ρ^0_{il} is the resistivity tensor of a QC in the absence of mechanical stresses, and $\rho^{(1)}_{il}$ and $\rho^{(2)}_{il}$ are the resistivity tensors induced by phonon and phason stresses, respectively. Thus the piezoresistance properties of QCs are described by the two tensors $P^{(1)}_{ilmn}$ and $P^{(2)}_{il\alpha n}$. In the case of icosahedral QCs, however, the piezoresistance properties can be characterized more conveniently by another two tensors $\Pi^{(1)}$ and $\Pi^{(2)}$ related to $\mathbf{P}^{(1)}$ and $\mathbf{P}^{(2)}$ by

$$P^{(1)}_{ilmn} = \rho^0_{ik} \Pi^{(1)}_{klmn} \qquad P^{(2)}_{il\alpha n} = \rho^0_{ik} \Pi^{(2)}_{kl\alpha n}. \tag{2}$$

Then equation (1) becomes

$$\mathcal{E}_i = \rho^0_{ik} (\delta_{kl} + \Pi^{(1)}_{klmn} T_{mn} + \Pi^{(2)}_{kl\alpha n} H_{\alpha n}) J_l. \tag{3}$$

Here $\mathbf{\Pi}^{(1)}(\mathbf{\Pi}^{(1)})$ and $\mathbf{\Pi}^{(2)}(\mathbf{P}^{(2)})$ are the piezoresistivity tensors induced by the phonon stress and the phason stress, respectively. Since in icosahedral QCs $\rho^0_{ik} = \rho^0 \delta_{kl}$, equation (3) has the form

$$\mathcal{E}_i = \rho^0 (\delta_{il} + \Pi^{(1)}_{ilmn} T_{mn} + \Pi^{(2)}_{il\alpha n} H_{\alpha n}) J_l. \tag{4}$$

Alternatively, the equation which describes the piezoresistance effect in QCs may be written in terms of the strains E_{pq} and $W_{\beta q}$

$$\mathcal{E}_i = \rho^0_{ik} (\delta_{kl} + m^{(1)}_{klpq} E_{pq} + m^{(2)}_{kl\beta q} W_{\beta q}) J_l, \tag{5}$$

where $m^{(1)}_{klpq}$ and $m^{(2)}_{kl\beta q}$ are the tensors of elastoresistive coefficients. From the generalized Hooke's law [11] it follows that the relations between $\mathbf{m}$ and $\mathbf{\Pi}$ are

$$\begin{aligned} m^{(1)}_{klpq} &= \Pi^{(1)}_{klmn} C_{mnpq} + \Pi^{(2)}_{kl\alpha n} R_{pq\alpha n} \\ m^{(2)}_{kl\beta q} &= \Pi^{(1)}_{klmn} R_{mn\beta q} + \Pi^{(2)}_{kl\alpha n} K_{\alpha n\beta q}. \end{aligned} \tag{6}$$

Table 1. Characteristics of 235 symmetry. (Note: $\tau = \frac{1}{2}(1+\sqrt{5})$.)

235	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$
Γ_1	1	1	1	1	1
Γ_3	3	τ	$1-\tau$	0	-1
Γ_3'	3	$1-\tau$	τ	0	-1
Γ_4	4	-1	-1	1	0
Γ_5	5	0	0	-1	1

Here **C**, **K** and **R** are elastic tensors of phonon field, phason field and phonon–phason coupling, respectively.

Since ρ_{ij}, T_{ij} and E_{ij} are all symmetric, in accordance with the symmetry of these tensors we have

$$\begin{aligned} &\Pi^{(1)}_{ilmn} = \Pi^{(1)}_{limn} = \Pi^{(1)}_{ilnm} = \Pi^{(1)}_{linm}, \qquad \Pi^{(2)}_{il\alpha n} = \Pi^{(2)}_{li\alpha n}, \\ &P^{(1)}_{ilmn} = P^{(1)}_{limn} = P^{(1)}_{ilnm} = P^{(1)}_{linm}, \qquad P^{(2)}_{il\alpha n} = P^{(2)}_{li\alpha n}. \end{aligned} \tag{7}$$

We can readily see that the number of the components of piezoresistivity tensor induced by the phonon stress ($\Pi^{(1)}_{ilmn}$ or $P^{(1)}_{ilmn}$) is 36, while the number of the piezoresistivity tensor induced by the phason stress ($\Pi^{(2)}_{il\alpha n}$ or $P^{(2)}_{il\alpha n}$) is 54 for 3D QCs and 36 for 2D QCs, respectively. However, additional restrictions arise from the point-group symmetry inherent in the QC considered and generally lead to a reduction in the number of independent components of the piezoresistivity (or elastoresistivity) tensor. In the following section we will discuss this point in detail.

3. Independent components of the piezoresistivity tensor

In this section we determine the number of independent components of the piezoresistivity tensor for all QCs. According to the higher-dimensional description of QCs a QC structure can be generated by projecting a higher-dimensional lattice (V) onto the physical space (V_E) where $V = V_E + V_I$ with V_I being the perpendicular space. Consequently, a vector in V_E transforms under the vector representation (Γ_A) of the symmetry group of the structure considered, whereas a vector in V_I transforms under another irreducible representation (Γ_B). Once the transformation properties of the vectors are specified, the physical property tensor of any rank can be determined by group representation theory. As an example, we consider the icosahedral QCs with 235 symmetry. This point group has five irreducible representations (cf table 1), one of which is one-dimensional (the identity representation), one four-dimensional, one five-dimensional and two are two-dimensional, respectively. In this case, $\Gamma_A = \Gamma_3$ and $\Gamma_B = \Gamma_3'$. Therefore, the components $\Pi^{(1)}_{ilmn}$ transform under

$$\begin{aligned} (\Gamma_3 \times \Gamma_3)_s \times (\Gamma_3 \times \Gamma_3)_s &= (\Gamma_1 + \Gamma_5) \times (\Gamma_1 + \Gamma_5) \\ &= 2\Gamma_1 + \Gamma_3 + \Gamma_3' + 2\Gamma_4 + 4\Gamma_5, \end{aligned} \tag{8}$$

where subscript s denotes the symmetric part of the direct product.

As is well known, the number of nonvanishing independent components of a physical property tensor is just the number of the identity representations which are contained in the direct product. From equation (8) it follows that there are two independent components of $\Pi^{(1)}_{ilmn}$ and one independent component of $\Pi^{(2)}_{il\alpha n}$. The transformation properties of $\Pi^{(1)}_{ilmn}$ ($\Pi^{(2)}_{il\alpha n}$) follow directly from those for $\rho^{(1)}_{il}$ ($\rho^{(2)}_{il}$) and T_{mn} ($H_{\alpha n}$). If we find the precise components of $\rho^{(1)}_{il}$ ($\Pi^{(2)}_{il\alpha n}$) and T_{mn} ($H_{\alpha n}$) that transform under the same constituent representations, we can construct all the invariants formed by their combinations, and then establish the independent

Table 2. Piezoresistance constants for 3D QCs. In this table the indices αn in the phason stress $H_{\alpha n}$ are arranged in the order 11, 22, 33, 23, 31, 12, 32, 31, 21. $\Pi_{44}^{(1)} = \frac{1}{2}(\Pi_{11}^{(1)} - \Pi_{12}^{(1)})$.

Point groups	Piezoresistance constants
235, $m\bar{3}\bar{5}$	$\mathbf{\Pi}^{(1)} = \begin{pmatrix} \Pi_{11}^{(1)} & \Pi_{12}^{(1)} & \Pi_{12}^{(1)} & 0 & 0 & 0 \\ \Pi_{12}^{(1)} & \Pi_{11}^{(1)} & \Pi_{12}^{(1)} & 0 & 0 & 0 \\ \Pi_{12}^{(1)} & \Pi_{12}^{(1)} & \Pi_{11}^{(1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Pi_{44}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{44}^{(1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{44}^{(1)} \end{pmatrix}_2$
	$\mathbf{\Pi}^{(2)} = \Pi_{111}^{(2)} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}_1$

components $\Pi_{ilmn}^{(1)}$ ($\Pi_{il\alpha n}^{(2)}$). Using the same method given in [9], we find that $\rho_{11}^{(1)} + \rho_{22}^{(1)} + \rho_{33}^{(1)}$ and $T_{11} + T_{22} + T_{33}$ transform under the same representation Γ_1 giving one invariant

$$(\rho_{11}^{(1)} + \rho_{22}^{(1)} + \rho_{33}^{(1)})(T_{11} + T_{22} + T_{33}). \tag{9}$$

Similarly, $(\frac{1}{\sqrt{6}}(\rho_{11}^{(1)} + \rho_{22}^{(1)} - 2\rho_{33}^{(1)}), \frac{1}{\sqrt{2}}(\rho_{11}^{(1)} - \rho_{22}^{(1)}), \sqrt{2}\rho_{12}^{(1)}, \sqrt{2}\rho_{31}^{(1)}, \sqrt{2}\rho_{23}^{(1)})$ and $(\frac{1}{\sqrt{6}}(T_{11} + T_{22} - 2T_{33}), \frac{1}{\sqrt{2}}(T_{11} - T_{22}), \sqrt{2}T_{12}, \sqrt{2}T_{31}, \sqrt{2}T_{23})$ transform under the same representation (Γ_5) giving another invariant

$$(\rho_{23}^{(1)}T_{23} + \rho_{31}^{(1)}T_{31} + \rho_{12}^{(1)}T_{12}) - (\rho_{11}^{(1)}T_{22} + \rho_{22}^{(1)}T_{33} + \rho_{11}^{(1)}T_{11}). \tag{10}$$

Thus, we obtain the corresponding nonvanishing components

$$\begin{aligned} &\Pi_{1111}^{(1)} = \Pi_{2222}^{(1)} = \Pi_{3333}^{(1)}, \qquad \Pi_{2323}^{(1)} = \Pi_{3131}^{(1)} = \Pi_{1212}^{(1)} = \tfrac{1}{2}(\Pi_{1111}^{(1)} - \Pi_{1122}^{(1)}), \\ &\Pi_{1122}^{(1)} = \Pi_{2211}^{(1)} = \Pi_{1133}^{(1)} = \Pi_{3311}^{(1)} = \Pi_{2233}^{(1)} = \Pi_{3322}^{(1)}. \end{aligned} \tag{11}$$

The piezoresistivity tensor for point group 235 can be also written in the matrix form

$$\mathbf{\Pi}^{(1)} = \begin{pmatrix} \Pi_{11}^{(1)} & \Pi_{12}^{(1)} & \Pi_{12}^{(1)} & 0 & 0 & 0 \\ \Pi_{12}^{(1)} & \Pi_{11}^{(1)} & \Pi_{12}^{(1)} & 0 & 0 & 0 \\ \Pi_{12}^{(1)} & \Pi_{12}^{(1)} & \Pi_{11}^{(1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Pi_{11}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{11}^{(1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{11}^{(1)} \end{pmatrix}_2 \tag{12}$$

where the subscript 2 stands for the number of independent components. The correspondences between the index pairs and single indices in physical space are, as usual,

$$\begin{aligned} (ij) &= 11 \quad 22 \quad 33 \quad 23 \quad 31 \quad 12 \\ i \ &= 1 \quad\ \ 2 \quad\ \ 3 \quad\ \ 4 \quad\ \ 5 \quad\ \ 6. \end{aligned} \tag{13}$$

The same method can be used for determining the phasonic piezoresistivity tensor $\mathbf{\Pi}^{(2)}$, but it should be noted that the phason variable ($\mathbf{w}$) transforms under another representation (Γ_3') and the components $\Pi_{il\alpha n}^{(2)}$ transform under

$$\begin{aligned} (\Gamma_3 \times \Gamma_3)_s \times (\Gamma_3 \times \Gamma_3') &= (\Gamma_1 + \Gamma_5) \times (\Gamma_4 + \Gamma_5) \\ &= \Gamma_1 + 2\Gamma_3 + 2\Gamma_3' + 4\Gamma_4 + 5\Gamma_5. \end{aligned} \tag{14}$$

Table 3. Piezoresistance constants for 2D QCs. In this table the indices αn in the phason stress $H_{\alpha n}$ are arranged in the order 11, 22, 23, 12, 13, 21. $P_{66}^{(1)} = \frac{1}{2}(P_{11}^{(1)} - P_{12}^{(1)})$.

Point groups	Piezoresistance constants
$5, \bar{5}, N,$ $\overline{N}, N/m$ $(N = 8, 10, 12)$	$\mathbf{P}^{(1)} = \begin{pmatrix} P_{11}^{(1)} & P_{12}^{(1)} & P_{13}^{(1)} & 0 & 0 & P_{16}^{(1)} \\ P_{12}^{(1)} & P_{11}^{(1)} & P_{13}^{(1)} & 0 & 0 & -P_{16}^{(1)} \\ P_{31}^{(1)} & P_{31}^{(1)} & P_{33}^{(1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{44}^{(1)} & P_{45}^{(1)} & 0 \\ 0 & 0 & 0 & -P_{45}^{(1)} & P_{44}^{(1)} & 0 \\ -P_{16}^{(1)} & P_{16}^{(1)} & 0 & 0 & 0 & P_{66}^{(1)} \end{pmatrix}_8$
$5m, 52, \bar{5}m,$ $Nmm, N22,$ $\overline{N}m2, N/mmm$ $(N = 8, 10, 12)$	$\mathbf{P}^{(1)} = \begin{pmatrix} P_{11}^{(1)} & P_{12}^{(1)} & P_{13}^{(1)} & 0 & 0 & 0 \\ P_{12}^{(1)} & P_{11}^{(1)} & P_{13}^{(1)} & 0 & 0 & 0 \\ P_{31}^{(1)} & P_{31}^{(1)} & P_{33}^{(1)} & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{44}^{(1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{44}^{(1)} & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{66}^{(1)} \end{pmatrix}_6$
$5, \bar{5}$	$\mathbf{P}^{(2)} = \begin{pmatrix} P_{111}^{(2)} & P_{111}^{(2)} & P_{123}^{(2)} & P_{112}^{(2)} & P_{113}^{(2)} & -P_{112}^{(2)} \\ -P_{111}^{(2)} & -P_{111}^{(2)} & -P_{123}^{(2)} & -P_{112}^{(2)} & -P_{113}^{(2)} & P_{112}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ P_{411}^{(2)} & -P_{411}^{(2)} & 0 & P_{412}^{(2)} & 0 & P_{412}^{(2)} \\ -P_{412}^{(2)} & P_{412}^{(2)} & 0 & P_{411}^{(2)} & 0 & P_{411}^{(2)} \\ 0 & P_{112}^{(2)} & 0 & -P_{111}^{(2)} & P_{123}^{(2)} & P_{111}^{(2)} \end{pmatrix}_6$
$5m, 52, \bar{5}m$	$\mathbf{P}^{(2)} = \begin{pmatrix} P_{111}^{(2)} & P_{111}^{(2)} & P_{123}^{(2)} & 0 & 0 & 0 \\ -P_{111}^{(2)} & -P_{111}^{(2)} & -P_{123}^{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ P_{411}^{(2)} & -P_{411}^{(2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{411}^{(2)} & 0 & P_{411}^{(2)} \\ 0 & 0 & 0 & -P_{111}^{(2)} & P_{123}^{(2)} & P_{111}^{(2)} \end{pmatrix}_3$
$N, \overline{N}, N/m$ $(N = 8, 10)$	$\mathbf{P}^{(2)} = \begin{pmatrix} P_{111}^{(2)} & P_{111}^{(2)} & 0 & P_{112}^{(2)} & 0 & -P_{112}^{(2)} \\ -P_{111}^{(2)} & -P_{111}^{(2)} & 0 & -P_{112}^{(2)} & 0 & P_{112}^{(2)} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ P_{112}^{(2)} & P_{112}^{(2)} & 0 & -P_{111}^{(2)} & 0 & P_{111}^{(2)} \end{pmatrix}_2$
$Nmm, N22,$ $\overline{N}m2, N/mmm$ $(N = 8, 10)$	$\mathbf{P}^{(2)} = \begin{pmatrix} P_{111}^{(2)} & P_{111}^{(2)} & 0 & 0 & 0 & 0 \\ -P_{111}^{(2)} & -P_{111}^{(2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -P_{111}^{(2)} & 0 & P_{111}^{(2)} \end{pmatrix}_1$
$12, \overline{12}, 12/m,$ $12mm, 1222,$ $\overline{12}m2, 12/mmm$	$\mathbf{P}^{(2)} = 0$

In this case $(\frac{1}{\sqrt{6}}(\rho_{11}^{(2)} + \rho_{22}^{(2)} - 2\rho_{33}^{(2)}), \frac{1}{\sqrt{2}}(\rho_{11}^{(2)} - \rho_{22}^{(2)}), \sqrt{2}\rho_{12}^{(2)}, \sqrt{2}\rho_{31}^{(2)}, \sqrt{2}\rho_{23}^{(2)})$ and $(H_{33}, \frac{1}{\sqrt{3}}(H_{11}+H_{22}+H_{13}), \frac{1}{\sqrt{3}}(H_{21}-H_{12}-H_{23}), \frac{1}{\sqrt{3}}(H_{11}+H_{31}-H_{22}), \frac{1}{\sqrt{3}}(H_{32}-H_{21}-H_{12}))$ transform under the same representation (Γ_5) giving one invariant

$$\begin{aligned}(\rho_{11}^{(2)} + \rho_{22}^{(2)} - 2\rho_{33}^{(2)})H_{33} + (\rho_{11}^{(2)} - \rho_{22}^{(2)})(H_{11} + H_{22} + H_{13})\\ + 2\rho_{12}^{(2)}(H_{21} - H_{12} - H_{23}) + 2\rho_{31}^{(2)}(H_{11} + H_{31} - H_{22})\\ + 2\rho_{23}^{(2)}(H_{32} - H_{21} - H_{12}).\end{aligned} \tag{15}$$

Then the corresponding nonvanishing components are

$$\begin{aligned}\Pi_{1111}^{(2)} &= \Pi_{1122}^{(2)} = \Pi_{1133}^{(2)} = \Pi_{1113}^{(2)} = -\Pi_{2211}^{(2)} = -\Pi_{2222}^{(2)}\\ &= \Pi_{2233}^{(2)} = \Pi_{2213}^{(2)} = -\tfrac{1}{2}\Pi_{3333}^{(2)} = -\Pi_{2312}^{(2)} = \Pi_{2332}^{(2)} = -\Pi_{2312}^{(2)}\\ &= \Pi_{3111}^{(2)} = -\Pi_{3122}^{(2)} = \Pi_{3131}^{(2)} = -\Pi_{1223}^{(2)} = -\Pi_{1212}^{(2)} = \Pi_{1221}^{(2)}.\end{aligned} \tag{16}$$

The corresponding matrix form is

$$\mathbf{\Pi}^{(2)} = \Pi_{111}^{(2)} \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0\\ -1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0\\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & -1\\ 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 1 \end{pmatrix}_1 \tag{17}$$

where $\Pi_{111}^{(2)}$ corresponds to $\Pi_{1111}^{(2)}$ since the latter has only first two indices in physical space.

Using this method we can determine the number of independent components of piezoresistivity tensor and their matrix forms for all other QCs. The results of 3D QCs and 2D QCs are given in tables 2 and 3, respectively.

4. Conclusion

In summary, we have investigated the piezoresistivity effect of QCs. It is found that the piezoresistance properties of QCs are more complicated than those of ordinary crystals because of the presence of the phason field. Several points can be noted:

(1) With regard to piezoresistivity tensor induced by the phonon stress, there are two classes in 2D QCs. One class consists of the QCs with the point groups 5, $\bar{5}$, N, $\overline{N}$ and N/m ($N = 8, 10, 12$), which have eight independent components $P_{ilmn}^{(1)}$. Another class consists of the QCs with the point groups $5m$, 52, $\bar{5}m$, Nmm, $N22$, $\overline{N}m2$ and N/mmm ($N = 8$, 10, 12), which have six independent components $P_{ilmn}^{(1)}$.

(2) With regard to the piezoresistivity tensor induced by the phason stress, there are five classes in 2D QCs. The first class consists of the QCs with the point groups 5, $\bar{5}$, which have six independent components $P_{il\alpha n}^{(2)}$. The second class consists of the QCs with the point group $5m$, 52 and $\bar{5}m$, which have three independent components $P_{il\alpha n}^{(2)}$. The third class consists of the QCs with the point groups N, $\overline{N}$, N/m ($N = 8, 10$), which have two independent components $P_{il\alpha n}^{(2)}$. The fourth class consists of the QCs with the point groups Nmm, $N22$, $\overline{N}m2$ and N/mmm ($N = 8, 10$), which have one independent component $P_{il\alpha n}^{(2)}$. The fifth class consists of the dodecagonal QCs, which have no such components.

(3) 3D QCs with 235 and $m\bar{3}\bar{5}$ symmetries have the same piezoresistance properties.

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